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A chiral superpropagator for pions

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Abstract. We consider the superpropagator constructed from a chiral $SU(2) \times SU(2)$ invariant Lagrangian. We show in general how the complications arising from the derivative couplings can be separated from the nonpolynomial, nonderivative part; and explicitly calculate the pion superpropagator for the unique choice of pion field that strict localizability requires.

In an Appendix we show that an extension of the Efimov–Fradkin Green function representation will give us n multiplet to n multiplet second order functions from the superpropagator.

1. Introduction

The use of nonlinear realizations of chiral symmetry leads us into writing invariant Lagrangians which are nonpolynomial functions of the fields. The usual 'effective' Lagrangian prescription would tell us to expand in a power series, take just the first few polynomial terms and then use the tree-graph approximation (Gasiorowicz and Geffen 1969 and references therein); but there is also interest in treating the Lagrangian as a more orthodox field theory object. This would involve putting in loops (Charap 1971) and then taking advantage of the fact that nonpolynomial Lagrangians may have renormalization advantages over most polynomial types (Salam 1971, Keck and Taylor 1971, Lehmann and Trute 1972).

We are interested here in chiral $SU(2) \times SU(2)$ and massless pions. The chiral invariant Lagrangian is nonpolynomial in the pion fields and thus the second order functions derived from it contain the exchange of any number of pions. The actual calculation of such objects is complicated by the fact that the interaction Lagrangian contains two derivatives and is a nonpolynomial function of a multiplet (triplet) of fields; thus the combinatorics arising from Wick's theorem become difficult. Delbourgo (1972) has shown that a straightforward extension of the usual integral-transform techniques will take care of the multiplets and we show here that we can also separate and handle the derivative complications for a certain class of second-order functions. These are the graphs that have n multiplets going to n multiplets, and can be obtained from the vacuum-to-vacuum graph, the so-called superpropagator, by taking derivatives with respect to propagators. In an Appendix we show how this follows from an extension of the Efimov–Fradkin representation of the Green functions (Efimov 1963, Fradkin 1963).

As far as the chiral invariance is concerned, the Lagrangian is arbitrary up to a redefinition of the pion field; but the desire for a 'good' field theory may give a preferred coordinate system. Certainly, if we apply Jaffe's (1967) condition of strict localizability, we get a unique pion field (Lehmann and Trute 1972) and this is the one we will use to calculate the superpropagator. We have previously used this choice, the exponential parametrization, to calculate the superpropagator appropriate to a pion–nucleon interaction with one derivative (Davies 1972).

In § 2 we give the general method for calculating a pion superpropagator, in § 3 we obtain a unique chiral Lagrangian and in § 4 we give the results of the superpropagator calculation. We conclude in § 5 with some brief remarks on the chiral SU(3) × SU(3) problem.

2. The general method for pion superpropagators

We will consider the simplest second-order function, the vacuum-to-vacuum superpropagator, as this contains all the multiplet and derivative complications and to some extent external lines can be taken into account by differentiating with respect to propagators. The work of Delbourgo (1972) on extending the Efimov–Fradkin Green function representation to include multiplets of fields, and that of Delbourgo *et al* (1969) on derivatively coupled scalar theories, has made advances towards this, and we indicate in Appendix 1 how the simplest combination of their results would allow us to get n multiplet to n multiplet graphs from the superpropagator.

We now describe our method for calculating the vacuum expectation value of the T^* product of two normally ordered, nonpolynomial functions of the pion fields and their derivatives.

Suppose we have the nonpolynomial function $\Lambda(\pi_i, \partial_\lambda \pi_j)^{ab\cdots}_{\mu\nu\cdots}$ where the $\mu\nu\ldots$ are uncontracted space–time labels and the $ab\ldots$ are uncontracted isospin indices. (By 'uncontracted' we mean uncontracted against other fields. The indices may, for instance, be contracted against numerical tensors but it is only the field dependence which interests us.) Further, suppose we want to calculate the object

$$\Sigma(\Delta)^{ab\ldots,pq\ldots}_{\mu\nu\ldots,\rho\lambda\ldots} = \langle 0|T^* :\Lambda(x)^{ab\cdots}_{\mu\nu\cdots}: :\Lambda'(y)^{pq\cdots}_{\rho\lambda\cdots}:|0\rangle \tag{1}$$

where the function Λ' may differ from Λ and the free field propagator Δ is given by

$$\langle 0|T(\pi_i(x)\pi_j(y))|0\rangle \equiv \langle \pi_i(x), \pi_j(y)\rangle \equiv \delta_{ij}\Delta(x-y).$$

We proceed in the following stages:

(i) Write Λ (and similarly Λ') in the form

$$\Lambda(\pi_i, \partial_\lambda \pi_j)^{ab\cdots}_{\mu\nu\cdots} = T(\pi)P(\pi_i, \partial_\lambda \pi_j)^{ab\cdots}_{\mu\nu\cdots} \tag{2}$$

where $T(\pi)$ is a nonpolynomial function of the isoscalar $\pi^2 \equiv \pi_i\pi_i$, and P is a polynomial in the field derivatives and those fields whose isopin indices are either uncontracted or contracted against the field derivatives.

(ii) Replace $T(\pi)$ by a triple Fourier transform representation

$$T(\pi) = \int_{-\infty}^{\infty} \mathrm{d}^3\xi \tilde{T}(\xi) \exp(\mathrm{i}\pi_j\xi_j) \tag{3}$$

so that all the field dependence is in the exponential (Delbourgo 1972). Then, if we expand the exponential as a power series, use Wick's theorem on the T^* product of (1), and resum, we have

$$\Sigma(\Delta)^{ab\ldots,pq\cdots}_{\mu\nu\ldots,\rho\lambda\ldots} = \int_{-\infty}^{\infty} \mathrm{d}^3\xi \int_{-\infty}^{\infty} \mathrm{d}^3\xi' \tilde{T}(\xi)\tilde{T}'(\xi') Q^{a\cdots}_{\mu\cdots} \exp(-\boldsymbol{\xi} . \boldsymbol{\xi}'\Delta) \tag{4}$$

where

$$Q^{a\ldots}_{\mu\ldots} = Q(\Delta, \Delta_\mu, \Delta_\nu, \Delta_{\mu\nu}, \ldots, \xi_i, \xi'_j)^{ab\ldots,pq\ldots}_{\mu\nu\ldots,\rho\lambda\ldots}$$

is a polynomial in the terms shown, its form depending on the P. The objects $\Delta_\nu, \Delta_{\mu\nu}, \ldots$ are given by

$$\begin{aligned} \Delta_\mu &\equiv \partial_\mu \Delta \\ \Delta_{\mu\nu} &\equiv \frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}\Delta(x-y) = -\partial_\mu\partial_\nu\Delta \end{aligned} \tag{5}$$

etc.

At this point we note that, as the fields in the T^* product are eliminated in pairs, then, as far as the isospin indices are concerned, $\Sigma(\Delta)^{ab\ldots,pq\ldots}$ can only be an SU(2) invariant numerical tensor built from Kronecker deltas like δ_{ap}. It then follows that

$$Q^{ab\ldots,pq\ldots} = \sum_r Q^{(r)}\alpha(r)^{ab\ldots pq\ldots} \tag{6}$$

where the $\{\alpha(r)\}$ are a complete, linearly independent set, each of the form $\delta_{ap}\delta_{bq}\ldots$. We can easily invert (6) to find the scalars $Q^{(r)}$ which will be of the form†

$$Q^{(r)} = Q^{(r)}(\Delta, \Delta_\mu, \Delta_\nu, \Delta_{\mu\nu}, \ldots, (\boldsymbol{\xi}\cdot\boldsymbol{\xi}'), (\boldsymbol{\xi}\cdot\boldsymbol{\xi}')^2, \boldsymbol{\xi}^2\boldsymbol{\xi}'^2, \ldots). \tag{7}$$

(iii) We now replace the ξ, ξ' dependence of Q by differential operators acting on the exponential integral and so separate all the dependence on the polynomials P and P'. We make the replacements

$$(\boldsymbol{\xi}\cdot\boldsymbol{\xi}')^n \to \left(-\frac{\partial}{\partial\Delta}\right)^n \tag{8a}$$

$$(\xi^2)^n(\xi'^2)^n \to \left(\frac{2}{\Delta}\frac{\partial}{\partial\Delta} + \frac{\partial^2}{\partial\Delta^2}\right)^n. \tag{8b}$$

The change (8*a*) is trivial and we prove (8*b*) in Appendix 2. If we make the above substitutions ($Q \to \hat{Q}$) we are simply left with

$$\Sigma(\Delta)^{ab\ldots,pq\ldots}_{\mu\nu\ldots,\rho\lambda\ldots} = \sum_r \alpha(r)^{ab\ldots pq\ldots}\hat{Q}^{(r)}(\Delta, \ldots, \partial/\partial\Delta, \ldots)_{\mu\nu\ldots,\rho\lambda\ldots}I(\Delta) \tag{9}$$

where

$$I(\Delta) = \int_{-\infty}^{\infty} d^3\xi \int_{-\infty}^{\infty} d^3\xi' \tilde{T}(\xi)\tilde{T}'(\xi')\exp(-\boldsymbol{\xi}\cdot\boldsymbol{\xi}'\Delta). \tag{10}$$

(iv) We now invert the relations (3) in order to rewrite (10) in terms of the original functions $T(u)$. Then, changing to polar coordinates and using the fact that the $T(u)$ are scalar functions of $u = \sqrt{u_i u_i}$ allows us to perform the angular integrals to arrive at

$$I(\Delta) = \frac{2}{\pi}\int_0^\infty du \int_0^\infty dv\, T(v)T'(i\Delta u)uv\sin(uv). \tag{11}$$

This, then, is the *only* integral to be performed, and is the one we would get if there were no field derivatives and no P terms. It is effectively this integral which is handling

† It will be important for (iii) that there is symmetry between ξ and ξ'. Nonsymmetric terms, such as $\xi^2(\xi'^2)^2$, could occur if P contained more fields than P', but we can always avoid this by transferring powers of π^2 from T' to P'.

the combinatorics of taking the two nonpolynomial functions together in the T product. In the next section we shall demand that the nonpolynomial functions T and T' be entire functions, thus there are no singularities to be encountered in the integration ranges of (11) at any finite points.

3. A unique chiral Lagrangian

According to whether we follow Isham (1969) or Gürsey (Chang and Gürsey 1967) we would write a chiral invariant Lagrangian for massless pions as

$$\mathscr{L}^{\mathrm{I}} = \tfrac{1}{2}g_{ij}(\pi)\partial_\mu \pi^i \partial_\mu \pi^j \tag{12a}$$

or

$$\mathscr{L}^{\mathrm{G}} = \tfrac{1}{4}F_\pi^2 \operatorname{Tr}(\partial_\mu \boldsymbol{U} \partial_\mu \boldsymbol{U}^\dagger). \tag{12b}$$

These two are well known to be equivalent, and arbitrary up to a redefinition of the pion field of the form $\pi_i \to \alpha(\pi^2)\pi_i$. What we shall do is demand that the Lagrangian gives a strictly localizable field theory in the sense of Jaffe (1967) which means that it must be an entire function of the fields of order less than 2. We find the Gürsey form (12b) most convenient for this analysis, and then having decided which pion field to use we can construct the interaction Lagrangian

$$\mathscr{L}_{\mathrm{int}} = \tfrac{1}{2}(g_{ij}(\pi) - \delta_{ij})\partial_\mu \pi^i \partial_\mu \pi^j \equiv v_{ij}(\pi)\partial_\mu \pi^i \partial_\mu \pi^j. \tag{13}$$

As we would expect, we get the same result as Lehmann and Trute (1972) who apply localizability to the same Lagrangian in a different form.

The unitary, unimodular Gürsey matrix $\boldsymbol{U}$ is a 2×2 matrix function of $\mathrm{i}\beta\boldsymbol{\pi}$, where $\beta = F_\pi^{-1}$ and $\boldsymbol{\pi} = \pi_i \boldsymbol{\sigma}_i$. The $\{\boldsymbol{\sigma}_i\}$ are the Pauli 2×2 traceless, hermitian matrices. Now, there are two common methods for parametrizing a unitary matrix, both involving a hermitian matrix $\boldsymbol{H}$. We have the exponential method $\mathrm{e}^{\mathrm{i}\boldsymbol{H}}$, and the Cayley, or rational method $(1+i\boldsymbol{H})(1-\mathrm{i}\boldsymbol{H})^{-1}$. If we further require unimodularity, then the exponential method requires $\boldsymbol{H}$ traceless, and for 2×2 matrices this is also true for the rational case. Thus we can write $\boldsymbol{H} = h_i \boldsymbol{\sigma}_i$ for some real $\{h_i\}$. Our problem is to parametrize $\boldsymbol{U}$ in terms of the pion fields, and as any SU(2) vector formed from the π_i can only be proportional to π_i we must have $h_i = \alpha(\pi)\pi_i$; where α is some arbitrary scalar function of $\pi = \sqrt{\pi_i \pi_i}$.

Thus we have $\boldsymbol{U} = \exp(\mathrm{i}\alpha(\pi)\boldsymbol{\pi})$ or

$$\boldsymbol{U} = (1+\mathrm{i}\alpha(\pi)\boldsymbol{\pi})(1-\mathrm{i}\alpha(\pi)\boldsymbol{\pi})^{-1} = \frac{1-\alpha^2\pi^2+2\mathrm{i}\alpha\boldsymbol{\pi}}{1+\alpha^2\pi^2}.$$

At this point we note that if we want entire (matrix) functions of the π_i we can best use the exponential method, and we must have α even. We also require the entire functions to be of order less than 2 and this means that α *can only be a constant*. This is because an entire function of the form $\exp(x^a)$ is only of order less than n if $a < n$.

Thus localizability requires us to use

$$\boldsymbol{U} = \exp(\mathrm{i}\beta\boldsymbol{\pi}). \tag{14}$$

Substituting this into (12b) and equating with (12a) gives

$$v_{ij}(\pi) = (1/2\beta^2)(\pi^2\delta_{ij} - \pi_i\pi_j)T(\pi) \tag{15}$$

where

$$T(\pi) = \pi^{-4}(\sin^2 \beta\pi - \beta^2\pi^2) \tag{16}$$

and this is the same as Lehmann and Trute (1972) obtain.

4. The superpropagator calculation

We now use the methods of § 2 to calculate the superpropagator:

$$\Sigma(\Delta) = \langle :\mathscr{L}_{\text{int}}(x):, :\mathscr{L}_{\text{int}}(y):\rangle \tag{17}$$

where $\mathscr{L}_{\text{int}}$ is the chiral interaction given by (13) and (15). We have no complications with uncontracted indices†, and the combinatorics give simply:

$$\hat{Q} = \Delta^4(\partial_\mu\partial_\nu \ln \Delta)^2\left(36+28\Delta\frac{\partial}{\partial\Delta}+4\Delta^2\frac{\partial^2}{\partial\Delta^2}\right) \tag{18}$$

where we have made the substitutions (8*a*) and (8*b*), and have used the formal identity

$$\Delta^2\partial_\mu\partial_\nu \ln \Delta = \Delta\partial_\mu\partial_\nu\Delta - \partial_\mu\Delta\partial_\nu\Delta.$$

In the integral (11) we have $T = T'$ given by (16), and if we define the dimensionless quantity $Z \equiv \beta^2\Delta$ we get the final result:

$$\Sigma(Z) = (1/8\beta^4)(\partial_\mu\partial_\nu \ln Z)^2 H(Z) \tag{19}$$

where

$$H(Z) = (1+8Z^2)C(Z) - 2Z\sinh(4Z) - \sinh^2(2Z) + 8Z^2$$

and

$$C(Z) \equiv 2\int_0^{2Z}\frac{\mathrm{d}x}{x}\sinh^2 x = \operatorname{chi}(4Z) - \ln(4\gamma Z).$$

We have checked to order Z^4 that the above expression agrees with that obtained directly from (17) by expanding $\mathscr{L}_{\text{int}}$ to low order in β.

Before leaving this section we should note that there are some unresolved problems concerned with the Feynman rules that we are using, that is, using T^* ordering with the interaction Lagrangian and renormalizing all tadpole contributions to zero by normal ordering, rather than using the canonical rules which involve T ordering and the interaction Hamiltonian. To second order in the interaction, which is the calculation that we perform, the work of Lazarides and Patani (1971) suggests that we are justified in using these naive Feynman rules, and preserve the Adler condition, provided that we interpret the distributions $\Delta_{\mu\nu}$, Δ_μ, etc in the analytically renormalized fashion that they prescribe. It has to be seen what relation such rules have to the canonical formalism (Gerstein *et al* 1971).

5. The chiral SU(3) × SU(3) problem

If we were interested in chiral SU(3) × SU(3) we would have formally the same Lagrangian as given by (12*a*) and (12*b*), we would just replace the π_i by the octet of pseudoscalar

† In a previous paper (Davies 1972) we outlined a calculation that involved nonpolynomial functions with uncontracted space–time and isospin labels. This was the case of a chiral invariant pion–nucleon interaction.

meson fields M_i. The problem of parametrizing the SU(3) Gürsey matrix is more complicated if we want the general solution (Barnes *et al* 1972) but applying strict localizability would once again determine the exponential parametrization.

Our general superpropagator method would proceed just as before, the triple Fourier transform (3) being replaced by an integral over eight scalar parameters, until we reached the equivalent of (10). The problem we now meet is that the SU(3) equivalents of the scalar functions T would in general be functions of the *two* SU(3) Casimir invariants. This means that we could not use the coordinate change from the cartesian M_i to eight-dimensional spherical polars with their single invariant; that is, the adjoint group SU(3)/Z(3) under which the M_i transform is isomorphic to a *subgroup* of the eight-dimensional rotation group SO(8). The SU(3) equivalent of the spherical polars (two invariants and six angles) with the associated Jacobian is known (Charap and Davies 1972) but so far has proved too complex to allow the integrals to be performed.

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Appendix 1

For a normally ordered interaction of scalar fields, $:\mathscr{L}(\phi):$, which may in general be nonpolynomial, we can write the Efimov–Fradkin (Efimov 1963, Fradkin 1963) representation of the n point Green function, to order N in the interaction, as:

$$S_{n_1 n_2 \ldots n_N}(x_1, x_2 \ldots x_N) = \exp\left(\frac{1}{2}\sum_{i\neq j} \Delta(x_i - x_j)\frac{\partial^2}{\partial\phi(x_i)\partial\phi(x_j)}\right)\prod_k \left(\frac{\partial}{\partial\phi(x_k)}\right)^{n_k} \mathscr{L}(\phi(x_k))\bigg|_{\phi=0} \tag{A.1}$$

where n_i is the number of external lines at the vertex x_i, and $n = \Sigma\, n_i$.

From now on we restrict our interest to second-order functions. For the case above we have

$$S_{n,m}(x, x') = \exp\left(\Delta(x-x')\frac{\partial^2}{\partial\phi\partial\phi'}\right)\left(\frac{\partial}{\partial\phi}\right)^n\left(\frac{\partial}{\partial\phi'}\right)^m \mathscr{L}(\phi)\mathscr{L}(\phi')\bigg|_{\phi=0} \tag{A.2}$$

and the generalization to Nth order will always be as simple as that from (A.2) to (A.1). We note that we can simply increase the number of external lines as:

$$\frac{\partial}{\partial\Delta}S_{n,m} = S_{n+1,m+1}\,.$$

If we now have a scalar interaction of an R dimensional multiplet of fields ϕ_a $(a = 1, 2, \ldots, R)$ we could generalize (A.2) (Delbourgo 1972) to get

$$S_{ab\ldots,pq\ldots} = \exp\left(\Delta\frac{\partial^2}{\partial\phi_c\partial\phi'_c}\right)\left(\frac{\partial}{\partial\phi_a}\cdots\right)\left(\frac{\partial}{\partial\phi'_p}\cdots\right)\mathscr{L}(\boldsymbol{\phi})\mathscr{L}(\boldsymbol{\phi}')\bigg|_{\phi=0} \tag{A.3}$$

where $a, b, \ldots$ are the multiplet components entering at vertex x, and we have used $\langle\phi_a, \phi'_b\rangle = \delta_{ab}\Delta$ to simplify the exponent. Note that differentiating with respect to Δ

again increases by one the external lines at each vertex; but does so in such a way that we get the sum over components of the multiplet.

Yet another generalization of (A.2) concerns interactions which involve scalar fields and their space–time derivatives (Delbourgo *et al* 1969, Hunt *et al* 1971). If we use the shorthand $\phi^\mu = \partial_\mu \phi$, we can write

$$S_{n,m} = \exp\left(\frac{\partial}{\partial \mathbf{\Phi}}\Delta\frac{\partial}{\partial \mathbf{\Phi}'}\right)\left(\frac{\partial}{\partial\phi}\cdots\frac{\partial}{\partial\phi^\mu}\cdots\right)\left(\frac{\partial}{\partial\phi'}\cdots\frac{\partial}{\partial\phi'^\nu}\cdots\right)\mathscr{L}(\phi,\phi^\mu)\mathscr{L}(\phi',\phi'^\nu)\Big|_{\phi=0} \qquad \text{(A.4)}$$

where

$$\frac{\partial}{\partial \mathbf{\Phi}}\Delta\frac{\partial}{\partial \mathbf{\Phi}'} \equiv \left(\frac{\partial}{\partial\phi},\frac{\partial}{\partial\phi'^\mu}\right)\begin{pmatrix}\Delta & \Delta_\nu \\ \Delta_\mu & \Delta_{\mu\nu}\end{pmatrix}\begin{pmatrix}\partial/\partial\phi' \\ \partial/\partial\phi'^\nu\end{pmatrix} \qquad \text{(A.5)}$$

and the Δ_μ etc are given by (5). To increase the number of external lines we could differentiate with respect to Δ, Δ_μ, Δ_ν or $\Delta_{\mu\nu}$ depending on what external lines we wanted to be derivatively coupled.

Now, in our chiral Lagrangian we have both derivatives *and* multiplets and so the simplest combination of (A.3) and (A.4) would be achieved by the replacement

$$\frac{\partial}{\partial \mathbf{\Phi}}\Delta\frac{\partial}{\partial \mathbf{\Phi}'} \to \frac{\partial}{\partial \mathbf{\Phi}_a}\Delta\frac{\partial}{\partial \mathbf{\Phi}'_a}$$

in (A.4), and by putting the appropriate multiplet labels on the external lines. In particular we can take the superpropagator

$$S_{0,0} = \exp\left(\frac{\partial}{\partial \mathbf{\Phi}_a}\Delta\frac{\partial}{\partial \mathbf{\Phi}'_a}\right)\mathscr{L}(x)\mathscr{L}(x')\Big|_{\phi=0} \qquad \text{(A.6)}$$

into any n multiplet to n multiplet second-order function by differentiation as we would for (A.4).

Appendix 2

Consider

$$J = \int_{-\infty}^{\infty} d^3\xi \int_{-\infty}^{\infty} d^3\xi' \tilde{T}(\xi)\tilde{T}'(\xi')\xi^2\xi'^2 \exp(-\boldsymbol{\xi}\,.\,\boldsymbol{\xi}'\Delta). \qquad \text{(A.7)}$$

Replacing ξ'^2 by $\Delta^{-2}(\partial/\partial\xi_i)^2$ acting on the exponential and using (3) gives

$$J = \Delta^{-2}\int_{-\infty}^{\infty} d^3\xi \tilde{T}(\xi)\xi^2(\partial/\partial\xi_i)^2 T'(\mathrm{i}\Delta\xi). \qquad \text{(A.8)}$$

Now T is a scalar function of $\xi = \sqrt{\xi_i\xi_i}$, so that if we use

$$\xi^2(\partial/\partial\xi_i)^2 T(\xi) = (2\xi\partial/\partial\xi + \xi^2\partial^2/\partial\xi^2)T(\xi)$$

and

$$\xi(\partial/\partial\xi)T(\Delta\xi) = \Delta(\partial/\partial\Delta)T(\Delta\xi)$$

$$\xi^2(\partial^2/\partial\xi^2)T(\Delta\xi) = \Delta^2(\partial^2/\partial\Delta^2)T(\Delta\xi)$$

we can write

$$J = \left(\frac{2}{\Delta}\frac{\partial}{\partial\Delta}+\frac{\partial^2}{\partial\Delta^2}\right)\int_{-\infty}^{\infty} d^3\xi \int_{-\infty}^{\infty} d^3\xi' \tilde{T}(\xi)\tilde{T}'(\xi') \exp(-\boldsymbol{\xi}\cdot\boldsymbol{\xi}'\Delta) \tag{A.9}$$

and (8*b*) follows directly from this.

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